

Phase Transitions in the Early Universe

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Abstract

It is believed that the temperature of the early Universe was once 300 GeV at 10^{-11} sec , or once 150 MeV at $3.3 \times 10^{-5}\text{ sec}$, and in fact even the hotter the earlier, e.g. 30 TeV at 10^{-15} sec . In this note, we are troubled by two basic questions: At very early times such as 10^{-15} sec , we worry about the matter densities much too higher such that the space-time and the matter may join to assume the meaning. As for the cosmological QCD phase and others, the matter density sounds to be quite normal but we learned from the (quantum) statistical mechanics that some phase transitions are first=order and so the latent heat (energy) in the case should play an important role.

At the times much earlier such as 10^{-15} sec or earlier, the standard theory gives the matter density so dense that the typical size of a hadron has to accommodate so many particles (by orders of magnitude more). At 10^{-15} sec , we have $T = 32\text{ TeV}$, $\rho_m = 3.2 \times 10^{18}\text{ gm/cm}^3$ and $\rho_\gamma = 6.4 \times 10^{30}\text{ gm/cm}^3$. (For comparison, the solar Schwarzschild density is $1.843 \times 10^{16}\text{ gm/cm}^3$ with $R_s = 2.953\text{ km}$ and the nuclear matter density is $2.0 \times 10^{14}\text{ gm/cm}^3$.) Thus, we suggest that we might reach the so-called "super-quantum" regime, in which the non-commutativity among the coordinates dictates. The suggestion of Snyder in 1947, though for different reasons, offers a nice formulation of the problem.

For the question about the importance of the latent heat (energy), we re-iterate the treatment of cosmological QCD phase transition and pinpoint where the latent heat eventually goes.

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1 Introduction

The discovery [1] of fluctuations or anisotropies, at the level of 10^{-5} , associated with the cosmic microwave background (CMB) has helped transformed the physics of the early universe into a main-stream research area in astronomy and in particle astrophysics, both theoretically and observationally [2]. CMB anisotropies[3] and polarizations[4], the latter even smaller and at the level of 10^{-7} , either primary (as imprinted on the last scattering surface just before the universe was $(379 \pm 8) \times 10^3$ years old) or secondary (as might be caused by the interactions of CMB photons with large-scale structures along the line of

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sight), are linked closely to the inhomogeneities produced in the early universe. Coupled with ESA's Planck Surveyor now in the orbit and other plans, we have a lot to anticipate.

Over the last four decades, on the other hand, the standard model of particle physics has been well established to the precision level of 10^{-5} or better in the electroweak sector, or to the level of $10^{-3} - 10^{-2}$ for the strong interactions. This gives the basis for describing the early Universe. In the theory, the electroweak (EW) phase transition, which endows masses to the various particles, and the QCD phase transition, which gives rise to confinement of quarks and gluons within hadrons in the true QCD vacuum, are two well-established phenomena. Presumably, the EW and QCD phase transitions would have taken place in the early universe, respectively, at around 10^{-11} sec and at a time between 10^{-5} sec and 10^{-4} sec, or at the temperature of about 300 GeV and of about 150 MeV, respectively. Indeed, it has become imperative to formulate the EW and QCD phase transitions in the early universe if a quantitative theory of cosmology can ever be reached.

The purpose of this note is to question what might be beyond our earlier investigations[5] of the cosmological QCD phase transition. In the earlier publication[5], we show the importance of the latent heat, basing on the MIT or Friedberg-Lee bag model. It was also pointed out that the latent "heat" or latent energy released, of course evolving into different forms of matter, could be linked with dark matter - argument by a simple arithmetic. We believe that the existence of the first-order phase transition(s) was there in the history of our Universe. In this note, we also worry about the roles of phase transitions in general in the much earlier Universe - what might be happening at 10^{-15} sec or earlier of our Universe when matter densities are unusually extremely high (so that quantum statistical mechanics in fact no longer applies).

2 The Background Universe in the Standard Description

A prevailing view regarding our Universe is that it originates from the joint making of Einstein's general relativity and the cosmological principle. Based upon the cosmological principle which state that our universe is homogeneous and isotropic, we use the Robertson-Walker metric to describe our Universe[6].

$$ds^2 = dt^2 - R^2(t) \left\{ \frac{dr^2}{1 - kr^2} + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \right\}. \quad (1)$$

Here the parameter k describes the spatial curvature with $k = +1, -1$, and 0 referring to an open, closed, and flat universe, respectively. The scale factor $R(t)$ describes the size of the universe at time t .

Assuming that the universe can be described by a perfect fluid, i.e., a fluid with the energy-momentum tensor $T^\mu{}_\nu = \text{diag}(\rho, -, -p, -p, -p)$ where ρ is the energy density and p the pressure (both as functions of t), then the Einstein equation, $G^\mu{}_\nu = 8\pi G_N T^\mu{}_\nu + \Lambda g^\mu{}_\nu$, gives rise to only two independent equations, i.e., from $(\mu, \nu) = (0, 0)$ and (i, i) components, The two equations can be combined to yield

$$\frac{\ddot{R}}{R} = -\frac{4\pi G_N}{3}(\rho + 3p) + \frac{\Lambda}{3}. \quad (2)$$

This last equation shows either that there is a positive cosmological constant or that $\rho + 3p$ must be somehow negative, if the major conclusion of the Supernovae Cosmology Project is correct [7], i.e. the expansion of our universe still accelerating ($\ddot{R}/R > 0$).

Alternately, assuming a simple equation of state, $p = w\rho$, we obtain, from the two independent equations,

$$2\frac{\ddot{R}}{R} + (1 + 3w)\left(\frac{\dot{R}^2}{R^2} + \frac{k}{R^2}\right) - (1 + w)\Lambda = 0, \quad (3)$$

which can be surprisingly solved for a constant w .

For example, let us assume for $t \geq T_0$ (T_0 a few times the age of our Universe) that it is dominated by the matter with a constant w_0 . We could write

$$R = e^{-at} + ce^{at}, \quad (4)$$

with the proportional constant fixed up if necessary. We obtain

$$a^2 = \Lambda, \quad c = \frac{k}{4a^2}. \quad (5)$$

Note that we try to fix the constants a^2 and c in the asymptotic solution.

It is interesting to note, in this asymptotic solution ($t \rightarrow +\infty$), that (1) w is dropped out completely, (2) Λ is better to be nonnegative (and very small), (3) to describe an expanding Universe, a should be negative, and (4) it is better to be the case that $ck \geq 0$. Asymptotically, this would be the solution as $t \rightarrow \infty$.

The above situation applies to $t \rightarrow \infty$ and NOT in the early Universe. For the early Universe, we may set Λ and k both to zero. Or, we have

$$2\ddot{R}R + (1 + 3w)\dot{R}^2 = 0, \quad \text{or, } R \propto t^n, \quad n = \frac{2}{3(1 + w)}. \quad (6)$$

The exception is at $w = -1$, for the inflation era. To describe the Inflation Era, we use $p = -\rho$ so that

$$\ddot{R} - \frac{\dot{R}^2}{R} = 0, \quad (7)$$

which has an exponentially growing, or decaying, solution $R \propto e^{\pm\alpha t}$, compatible with the so-called "inflation" or "big inflation". In fact, considering the simplest case of a real scalar field $\phi(t)$, we have

$$\rho = \frac{1}{2}\dot{\phi}^2 + V(\phi), \quad p = \frac{1}{2}\dot{\phi}^2 - V(\phi), \quad (8)$$

so that, when the "kinetic" term $\frac{1}{2}\dot{\phi}^2$ is negligible, we have an equation of state, $p \sim -\rho$. In addition to its possible role as the "inflaton" responsible for inflation. Such field has also been invoked to explain the accelerating expansion of the present universe, as dubbed as "quintessence" or "complex quintessence"[8].

Let's look at the standard textbook argument leading to the radiation-dominated universe and the matter-dominated universe:

For the **Radiation-Dominated Universe**, we have $p = \rho/3$. For simplicity, we assume that the curvature is zero ($k = 0$) and that the cosmological constant is negligible ($\Lambda = 0$). In this case, we find from Eq. (6)

$$R \propto t^{\frac{1}{2}}. \quad (9)$$

Another simple consequence of the homogeneous model is to derive the continuity equation:

$$d(\rho R^3) + p d(R^3) = 0. \quad (10)$$

Accordingly, we have $\rho \propto R^{-4}$ for a radiation-dominated universe ($p = \rho/3$) while $\rho \propto R^{-3}$ for a matter-dominated universe ($p \ll \rho$). The present universe is believed to have a matter content of about 5%, or of the density of about $5 \times 10^{-31} g/cm^3$, much bigger than its radiation content $5 \times 10^{-35} g/cm^3$, as estimated from the 3° black-body radiation. However, as $t \rightarrow 0$, we anticipate $R \rightarrow 0$, extrapolated back to a very small universe as compared to the present one. Therefore, the universe is necessarily dominated by the radiation during its early enough epochs.

For the radiation-dominated early epochs of the universe with $k = 0$ and $\Lambda = 0$, we could deduce,

$$\rho = \frac{3}{32\pi G_N} t^{-2}, \quad T = \left\{ \frac{3c^2}{32\pi G_N a} \right\}^{\frac{1}{4}} t^{-\frac{1}{2}} \cong 10^{10} t^{-1/2} (^\circ K). \quad (11)$$

These equations tell us a few important times in the early universe, such as $10^{-11} sec$ when the temperature T is around $300 GeV$ during which the electroweak (EW) phase transition is expected to occur, or somewhere between $10^{-5} sec$ ($\cong 300 MeV$) and $10^{-4} sec$ ($\cong 100 MeV$) during which quarks and gluons undergo the QCD confinement phase transition.

For the **Matter-Dominated Universe**, we have $p \approx 0$, together with the assumption that $k = 0$ and $\Lambda = 0$. Eq. (6) yields

$$R \propto t^{\frac{2}{3}}. \quad (12)$$

As mentioned earlier, the matter density ρ_m scales like R^{-3} , or $\rho_m \propto t^{-2}$, the latter similar in the radiation-dominated case.

When $t = 10^9 sec$, we have $\rho_\gamma = 6.4 \times 10^{-18} gm/cm^3$ and $\rho_m = 3.2 \times 10^{-18} gm/cm^3$, which are close to each other and it is almost near the end of the radiation-dominated universe. The present age of the Universe is 13.7 billion years - for a large part of it, it is matter-dominated although now we have plenty of dark energy (65% \sim 70%).

However, it is generally believed that our present universe is already dominated by the dark energy (the simplest form being of the cosmological constant; about 70%) and the dark matter (about 25%). The question is when this was so - when the dark part became dominant.

In other words, the proper language should include (1) the radiation density, (2) the matter density, (3) the dark-matter density, and (4) the dark-energy density - and it determines the evolution of the universe. Apparently, the entry of dark matter and of dark energy only makes the study of evolution of the Universe much more interesting.

3 Beyond the Standard Limits

What do we anticipate at TeV energies or higher? We know that the electrons and other building blocks of matter all look like point-like particles, or still look like point-like particles when the resolutions are 10^{-17} centimeters or better. In the range near $10^{-8} cm$, we have quantum mechanics and we incorporate it into the equation of state (EoS) via quantum statistical mechanics.

What do we have at TeV energies? There are many different scenarios - maybe we will consider two options in this note. Option No.1 is the minimal Standard Model. Option No.2 is the minimal family gauge theory coupled with the minimal Standard Model ("mfSM") [9]. In Option (1), the dark matter has to come from the model itself, such as from the latent energy [5]. In Option (2), there is a sector called "the dark-matter world", which couples very weakly with the ordinary-matter world. In either scenario, we assume the least to be seen.

The other important thing has to do with that we have to understand a lot more by going to the extremely high matter densities. In what follows, we suspect that we are hitting the so-called "**super-quantum**" regime. Let us try to explain the idea. (We call it "super-quantum" or "sub-quantum"; we prefer to use "super-quantum" from now on.)

Suppose that the formulae in the last section are correct; then at time $t = 10^{-15} sec$ we could determine T , ρ_m , and ρ_γ as follows:

$$T = 32 TeV : \quad \rho_m = 3.2 \times 10^{18} gm/cm^3, \quad \rho_\gamma = 6.4 \times 10^{30} gm/cm^3. \quad (13)$$

To get the feeling of these numbers, suppose that we stack the entire Solar mass into a small sphere that it becomes a black hole: $\rho_m = 1.843 \times 10^{16} gm/cm^3$ at $R_s = 2.953 km$. The nuclear matter density ρ_N is $2 \times 10^{14} gm/cm^3$.

So, at time $10^{-15} sec$, the matter density $3.2 \times 10^{18} gm/cm^3$ is already many orders of magnitude denser (or higher). But this is what is predicted by our standard theory (i.e. the Friedman-Robertson-Walker metric and the equation of state for ordinary matter). Of course, the radiation density (the boson's density) is even more ridiculously higher.

At this point, we have no ideas about ρ_{DM} ; presumably it all depends on how the majority of dark matter gets manufactured. On the dark energy, it would be tiny if it is indeed described by the cosmological constant.

Let us try to simplify our thinking for the moment. If we are talking about "point-like" objects, such as electrons or quarks, there might be a characteristic size scale, called as " r_0 ". We believe that the trouble begins when the distance is much smaller than r_0 - for instance, the standard hypothesis for the quantum statistical mechanics, thus for the derivation of the equation of state, no longer holds. For the matter at $T = 32 TeV$ or $\rho_m = 3.2 \times 10^{18} cm^{-3}$, we should worry about the presence of r_0 . In other words, the physical "point-like" "particle/matter" assumes its meaning with the space-time - it could be different from the ideological mathematical "point-like". (In other words, the physical "point-like" and space-time joins together to have a meaning, unlike the mathematical "point-like".)

Maybe at this juncture we could remind ourselves the so-called "quantum regime", where the momenta \vec{p} do not commute with the coordinates \vec{x} . The quantum regime is relevant when we deal with ordinary atomic or subatomic scale (or, distances). When the

Universe was 10^{-15} second old or younger, we are dealing with sizes much smaller than the hadron sizes - it is safe to introduce the "super-quantum regime": At these scales, the basic variables are still the coordinates (\vec{x}, t) but we could generalize them by making them non-commuting, or operators. (What else?)

The presence of the quantum regime at the atomic or subatomic scale suggests the "existence" of the "super-quantum" regime, perhaps $10^{-18} cm$ away. Again, the concept could be different from the ideological mathematical "point-like". In fact, the thoughts are similar to those suggested from a lot of physicists, famous or not. It is the very fundamental difference between the physicists and the mathematicians, such as the meaning of "point-like" and others.

Why do we have to worry about the super-quantum regime well ahead of the Planck era? To say that from the Planck length $1.616 \times 10^{-33} cm$ to $10^{-18} cm$ ($10^{-5} fm$), or from the Planck time $10^{-43} sec$ to $10^{-15} sec$, there is nothing (desert) - nobody would believe in it. In other words, we should worry about the "super-quantum" era if we analyze the "size" and the matter together and the "contradiction" (or the paradox) that we have.

The "size" of an object, such as an electron or a hadron, is not a well-defined concept. LEP at CERN allowed us to accelerate the electrons to $100 GeV$, or let us probe $10^{-6} Fermi$ without the need to take into account the "size" of the electron. But how small could we go in this direction? For hadrons, the "size" is dictated by the strong interactions, or QCD. Analogously, maybe the "size" of an electron is dictated by the electroweak theory, in the TeV regime. Maybe the "super-quantum" regime could be set in after that, or orders after that, but definitely well before the Planck scale.

It is well-known from the textbook of (quantum) statistical mechanics that the scale or distance for the spacing of particles should be larger than the intrinsic distance of the individual particle (or the de Broglie wavelength of the molecule) - clearly, we eventually would go to the regime that this very basic fact would be contradicted in defining the quantum statistical mechanics. In other words, we have nothing to "derive" the equation of state (EoS), to be used together with the Einstein equation (on general relativity, something similar). In our opinion, we should face this difficult situation instead of thinking of that we already have solved a complete set of equations for cosmology.

What would be the algebra, or the graded Lie algebra, among (x, y, z) or among (x, y, z, ict) ? Let's try to argue in the following way. In going from the "classical" regime to the "quantum" regime, we make the quantities (\vec{x}, \vec{p}) noncommutative and thus obtain the operator algebra. Now in the "quantum" regime, the basic variables are (x, y, z, ct) and the most general, and perhaps the only, way in the "super-quantum" regime is trying to introduce some noncommutative algebra among these variables (x, y, z, ct) .

As mentioned earlier, the matter density is predicted to be $3.2 \times 10^{18} gm/cm^3$ when the Universe was $10^{-15} sec$. This means that a volume of one hadron size (judging from the nuclear matter density) would accommodate 16,000 hadrons or matter particles. (Some people wouldn't worry about the situation until the Planck time or the Planck distance is reached.) Our thinking is as follows: As we go to smaller and smaller such that a volume of one hadron size has to accommodate thousands or more hadrons, the size by itself does not assume the meaning and instead the size and the matter may jointly have the meaning. The size without the matter, or the matter without the size (defined through its space-time), maybe lacks the (physics) meaning.

In the simple case, we need to take the coordinates (x, y, z, ct) as from the operators and to the first approximation to implement Lorentz symmetry in some way. Since Lorentz invariance is known to be true to our experimental accuracy, we are lucky to have this important guideline. The next question is to define the "operations" among \hat{x} , \hat{y} , \hat{z} , and \hat{ct} such that d^2s and others have their meanings.

We could put more thoughts along this line. Let (x, y, z, ct) be the Cartesian variables, which we need to label the coordinates (operators). The coordinates $(\hat{x}, \hat{y}, \hat{z}, \hat{ct})$ may be regarded as mappings from R^4 to C^4 (or something else). The next thing is to implement Lorentz symmetry - to define the ten operators, i.e. the Hamiltonian H , the three momentum \vec{P} , the three angular momentum \vec{J} , and the three Lorentz-boost operators \vec{K} or \vec{M} . These ten operators should be the mappings from the coordinates (operators themselves). Fortunately, the idea was already carried out early on by H. Snyder[10], for a different basic motivation.

In other words [10], \hat{x} , \hat{y} , \hat{z} , and \hat{t} are hermitian operators for the space-time coordinates of a particular Lorentz frame; the operators \hat{x} , \hat{y} , \hat{z} , and \hat{t} are such that the spectra of the operators \hat{x}' , \hat{y}' , \hat{z}' , \hat{t}' formed by taking linear combinations of \hat{x} , \hat{y} , \hat{z} , \hat{t} , which leaves invariant the quadratic form, $s^2 = x^2 + y^2 + z^2 - c^2t^2$, shall be the same as the spectra of \hat{x} , \hat{y} , \hat{z} , and \hat{t} .

To find operators \hat{x} , \hat{y} , \hat{z} , and \hat{t} possessing Lorentz invariant spectra, we may consider the homogeneous quadratic form[10]:

$$\eta^2 = -\eta_0^2 + \eta_1^2 + \eta_2^2 + \eta_3^2 + \eta_4^2, \quad (14)$$

in which the η 's are assumed to be real variables and they may be regarded as the homogeneous projective coordinates of a real four-dimensional space of constant curvature (a De Sitter space).

We may define[10]

$$\hat{x} = ia(\eta_4\partial/\partial\eta_1 - \eta_1\partial/\partial\eta_4), \quad (15)$$

and analogously for \hat{y} , \hat{z} , and \hat{ct} , where a is some unit length (defined in the super-quantum regime; it was the nature unit in the Snyder's discrete space-time). Accordingly, we introduce three angular-momentum operators and three Lorentz-boost operators as follows:

$$L_x = i\hbar(\eta_3\partial/\partial\eta_2 - \eta_2\partial/\partial\eta_3) \quad (16)$$

and analogously for L_y and L_z .

$$M_x = i\hbar(\eta_0\partial/\partial\eta_1 + \eta_1\partial/\partial\eta_0) \quad (17)$$

and similarly for M_y and M_z . Thus, we have

$$[\hat{x}, \hat{y}] = (ia^2/\hbar)L_z, \quad [\hat{ct}, \hat{x}] = (ia^2/\hbar)M_x, \quad (18)$$

and so on (noncommutative geometry in terms of six relations).

There may exist an interesting connection for this De Sitter space. We know that the cosmology written on the 4-dimensional hyper-surface $-\eta_0^2 + \eta_1^2 + \eta_2^2 + \eta_3^2 + \eta_4^2 = \alpha^2$ would possess the cosmological constant $\Lambda = 3/\alpha^2$. In the Snyder's language the interchange

$\eta_4 \rightarrow \eta$ is in fact optional. Therefore, we have the most basic "prediction" - the cosmological constant Λ is connected with the physics of the "super-quantum" era. Now we have [11]

$$\rho_c \Omega_\Lambda = 7.20565 \times 10^{-30} \text{ gm} \cdot \text{cm}^{-3} = 4.0421 \times 10^{-6} (\text{GeV}/c^2) \cdot \text{cm}^{-3}, \quad (19)$$

which explains the current dark energy density (74% of the critical energy density). Equating this to $\Lambda/(8\pi G_N)$, we obtain

$$\Lambda = 1.2087 \times 10^{-35} \text{ sec}^{-2}. \quad (20)$$

Hence, we have

$$\alpha^2 = 3/\Lambda = 2.4820 \times 10^{35} \text{ sec}^2, \quad \text{or} \quad \alpha = 4.98 \times 10^{17} \text{ sec}. \quad (21)$$

It is quite amazing to be close to the age of our Universe: 13.69 Gyr ($= 4.3202 \times 10^{17} \text{ sec}$). Since what we do is to specify a four-dimensional surface in a de Sitter space, these numbers could be taken as the consistency of these thoughts.

Surprisingly enough, the value of de Sitter α as derived from the tiny Λ (previously coming from nowhere) is remarkably close to the age of our Universe - this might take as the clue to resolve the problems of small cosmological constant, etc.

Our discussion indicates that there are other ways to introduce the noncommutative algebra [10, 12] but there are reasons to try out the Snyder's option. We could pursue further by making the choice of the noncommutative algebra along this line and we could write, for the space-time coordinates X_μ as operators,

$$[X_\mu, X_\nu] = \theta_{\mu\nu} = i\lambda^{-2} c_{\mu\nu}, \quad (22)$$

with $c_{\mu\nu}$ assumed to be a real antisymmetric matrix with elements of order one which commute with the space-time coordinates.

Of course, there are different consistent choices of $\theta_{\mu\nu}$ but the Lorentz symmetry insisted in the Snyder's language should play the fundamental role. As for the inverse size λ , it is time to try out the range $(1 - 100) \text{ TeV}$ in view of LHC,

What do we have different options? In the well-known transition from the classical to the quantum era, either momenta or coordinates get eliminated for making them operators. Now to obtain the "super-quantum" era, the coordinates become noncommutative - the way to recognize the operator nature. The uniqueness nature of the procedure is rather transparent, though there might be some alternates in the mathematics.

Noting parenthetically, noncommutative QED is the primary suspect [13, 14] since photons and the other building blocks (quarks, leptons, etc.), very dense of them are at stake. In other words, there are good reasons to believe that for our Universe at 10^{-15} sec or similar the rule of the game becomes the noncommutative coordinates, so that noncommutative QED, noncommutative QCD, and the noncommutative Standard Model dictates everything. Calculations similar to ref.[14] now finds a place to make sense.

We don't know what "point-like particles" means to us. Maybe some sort of "effective size", for 10^{-20} cm or smaller, could make sense. If we have the "effective" size, then putting two or more particles in the volume of one particle should be prohibited or obstructed.

The other reason is when we go from 1 cm to 10^{-8} cm we have quantum mechanics - in comparison, when we go from 10^{-8} cm to 10^{-20} cm , we might be hitting the "super-quantum" regime, or eventually. Among many possibilities, we think that the Snyder's option [10], which goes over the fifth dimension (de Sitter space) to label the coordinates (classically) and then maintains Lorentz symmetry while making the four-dimensional space-time discrete, is the most interesting. We should pursue the Snyder's option, at the "right" scales, much further. We motivate ourselves from the consideration of phase transitions, the matter systems that are unusually dense and challenge the notion of space-time.

Maybe we should simplify our arguments as follows: If we go smaller from the quantum era (around 10^{-8} cm) to the super-quantum era (around $10^{(-20)}\text{ cm}$ or smaller), the main fundamental question is whether the super-quantum era exists. The scale of 10^{-20} cm is about right for such change. Then, the non-commutativity for the coordinates is "must" if we think about it. On the other hand, if we try to think about the "point-like" nature of the electron, for a good example, the limitation on the scale is also about 10^{-20} cm .

If we think more and more, it is of utmost importance to look for the small scale of the super-quantum era. We shouldn't talk about quantum statistical mechanics when it is clear that it does not make sense any more - just look at that the particle separation is becoming much smaller than the effective distance of the inter-particle interactions (against the basic ansatz of the statistical mechanics). We simply cannot push the knowledge (the science) way beyond the limits of its validity.

4 A Question on the Cosmological QCD Phase Transition

In an early paper[15], we present our thoughts about the cosmological QCD phase transition. It happened at between 10^{-5} sec and 10^{-4} sec , when the densities appear to be "normal" for the hadron matter. That is, we don't have to worry about "super-quantum" physics. However, the real story about the QCD phase transition is rather complicated but may be truly important for phase transitions in the early Universe. The question in mind has to do with the latent heat (energy) of the phase transition of first-order - similar to the zero-point energy, or the Cosmological Constant, or that, when we differentiate, it disappears; supposedly that it is no longer relevant but it is NOT TRUE.

To describe the cosmological QCD phase transition, it is unlikely that we would wait for until we have solved QCD itself - often a formidable task. In this case, we could use bag models as the first approximation but maybe the phase transition in question is described as the first-order transition while in reality a second-order one. Then, solving QCD exactly becomes very important. Of course, we all know that the MIT bag model[16] or the Friberg-Lee non-topological soliton model[17] invokes the bag constant, or the "zero-point" energy, which implies the first order for phase transition.

At the temperature $T > T_c \sim 150\text{ MeV}$, i.e., before the phase transition takes place, free quarks and gluons can roam anywhere. As the Universe expands and cools, eventually passing the critical temperature T_c , the bubbles nucleate here and there. These bubbles "explode", as we call it "exploding solitons" (or "low-temperature solitons"). When it reaches the "supercooling" temperature, T_s , or something similar, the previous bubbles become too many and in fact most of them become touched each other - now the false vacua or "bubbles" of different kind (where quarks and gluons can move freely) start to

collapse - or we call it "imploding solitons" (or "high-temperature solitons"). When all these bubbles of different kind implode completely, the phase transition is now complete.

The "imploding" solitons with boundaries should find a way to "glue" together, such as the formation of domain walls, vortices, etc., sometimes with nontrivial topology.

There is some specialty regarding the cosmological QCD phase transition. Namely, the collapse of the false vacuum does depend on the inside quark-gluon content - e.g., if we have a three-quark color-singlet combination inside, the collapse of the false vacuum would stop (or stabilize) at a certain radius (we called the bag radius, like in the MIT bag radius); of course, there are meson configurations, glueballs, hybrids, six-quark or multi-quark configurations, etc. The cosmological QCD phase transition does not eliminate all the false vacua; rather, the end state of the transition could have at least lots of baryon or meson states, each of them has some false vacuum to stabilize the system.

How big can a bubble grow? It is with the fastest speed which the bubble can grow is through the speed of light or close to the speed of light. The bubble could sustain from the moment it creates, say, $T \approx T_c$ to the moment of supercooling, $T_s \sim 0.95 \cdot T_c$, or during the time span $t \sim 3 \times 10^{-5} \times 0.05 \text{ sec}$ (or $1.5 \times 10^{-7} \text{ sec}$). So, the bubble can at most grow into $c \cdot 1.5 \times 10^{-7} \text{ sec}$ or $4.5 \times 10^3 \text{ cm}$.

How many ("low-temperature") bubbles are there of the entire Universe when the space were filled up by the bubbles (when the phase transition was complete)? The point is that two bubbles are separated by the domain wall of certain structure (with some energy deposited in there - some surface energy). The domain walls cannot disappear completely - not only sometime because of the possible nontrivial topology but that there should be some QCD dynamics to annihilate the walls.

As a yardstick, we note that, at $t \sim 10^{-5} \text{ sec}$ or $T \sim 300 \text{ MeV}$, we have

$$\rho_\gamma \sim 6.4 \times 10^{10} \text{ gm/cm}^3, \quad \rho_m \sim 3.2 \times 10^3 \text{ gm/cm}^3. \quad (23)$$

Or, slightly later when QCD phase transition has completed, at $t \sim 10^{-4} \text{ sec}$ or $T \sim 100 \text{ MeV}$, we have

$$\rho_\gamma \sim 6.4 \times 10^8 \text{ gm/cm}^3, \quad \rho_m \sim 1.0 \times 10^2 \text{ gm/cm}^3. \quad (24)$$

When the low-temperature bubbles fill up the space, the neighboring two bubbles would in general be labelled by different $\theta_{i,j}$ representing different but degenerate vacua - we assume that there are infinite many choices of θ ; they are degenerate but complete equivalent. As example, see [5, 15]. Note that the internal structure of QCD is complicated enough to disfavor the non-degeneracy of the vacuum. The domain wall is used to separate the two regions. Three different regions would meet in a line - which we call a vortex. We have to estimate the total energy associated with the domain walls and the vortices - particularly when these objects persist to live on for a "long" time - say, $\tau \gg 10^{-4} \text{ sec}$. These domain walls and vortices are governed, in the QCD phase transition in the early Universe, by the QCD dynamics.

We may start with a simple estimate - the expansion factor since the QCD phase transition up to now. The present age of the Universe is 13.7 billion years or $13.7 \times 10^9 \times 365.25 \times 24 \times 3600$ or 4.323×10^{17} seconds. About the first 10^9 sec period of the hot big bang is previously-believed radiation-dominated. Consider the length 1.0 fermi at $t \sim 10^{-5} \text{ sec}$, it

will be expanded by a factor of 10^7 up to $t \sim 10^9 \text{ sec}$ (radiation-dominated) and expanded further by another factor of 5.7×10^5 until the present time - so, a total expansion factor of 5.7×10^{12} ; changing a length of 2 fermi at $t \sim 10^{-5} \text{ sec}$ into a distance of 1 cm now. A proton presumably of $R = 1 \text{ fermi}$ at $t \sim 10^{-5} \text{ sec}$ should be more or less of the same size now; or, the bag constant or the energy associated with the false vacuum should remain the same.

What would happen to the pasted or patched domain walls as formed during the cosmological QCD phase transition? According to [5, 15], we note that the solutions in previously two different true-vacuum regions cannot be matched naturally - unless the K values match accidentally. But it is clear that the system cannot be stretched or over-stretched by such enormous factor, 10^{12} or 10^{13} . I believe that the field ϕ , being effective, cannot be lonely; that is, there are higher-order interactions such as

$$c_0 \phi G_\mu^a G^{\mu,a}, \quad c_1 \phi GGG, \dots, \quad d_0 \phi \bar{\psi} \psi, \quad (25)$$

some maybe being absent because of the nature of ϕ . In other words, we may believe that the strong interactions are primarily responsible for the phase transition in question, such that the effective field ϕ couples to the gluon and quark fields; the details of the coupling are subject to further investigations.

That is, when the field ϕ responsible for the pasted or patched domain walls is *effective* - the ϕ field couples, in the higher-order (and thus weaker) sense, to the gluon and quark fields. It is very difficult to estimate what time is needed for pasted domain walls to disappear, if there are no nontrivial topology involved. If there is some sort of nontrivial topology present, there should left some kind of topological domain nugget - however, energy conservation should tell us that it cannot be expanded by too many orders.

In other words, the energy associated with the cosmological QCD phase transition, mainly the vacuum energy associated with the false vacuum, disappeared in several ways, viz.: (1) the bag energies associated with the baryons and all the other color-singlet objects, (2) the energies with all kinds of topological domain nuggets or other topological objects, and (3) the decay products from pasted or patched domain walls with trivial topology.

Considering just before the critical temperature $T = T_c \approx 150 \text{ MeV}$ or $t \approx 3.30 \times 10^{-5} \text{ sec}$, we have

$$\rho_{vac} = 1.0163 \times 10^{14} \text{ gm/cm}^3, \quad \rho_\gamma = 5.88 \times 10^9 \text{ gm/cm}^3, \quad \rho_m = 6.51 \times 10^2 \text{ gm/cm}^3. \quad (26)$$

Here the first term is what we expect the system to release - the so-called "latent heat"; I call it "latent energy" for obvious reasons. The identification of the latent "heat" with the bag constant is well-known in MIT and Coulomb bag models [16, 17].

This can be considered just before the cosmological QCD phase transition which took place.

As time went on, the Universe expanded and the temperature cooled further - from the critical temperature to the supercooling temperature ($T_s \sim 0.95 \times T_c$ with the fraction 0.95 in fact just a common estimate) and even lower, and then the cosmological QCD phase transition was complete. When the phase transition was complete, we should estimate how the energy ρ_{vac} is to be divided.

Let's assume that the QCD phase transition was completed at the point T_s (in fact maybe a little short after T_s). Let's take $T_s = 0.95 T_c$ for simplicity. We would like to know

how the energy ρ_{vac} is to be divided. First, we can estimate those remained with the baryons and other color-singlet objects - the lower limit is given by the estimate on the baryon number density: $\rho_m = 6.51 \times 10^2 gm/cm^3$ or $3.65 \times 10^{26} GeV/c^2/cm^3$. So, in the volume $1.0 cm^3$ or $10^{39} fermi^3$, we have at least 3.65×10^{26} baryons. One baryon has the volume energy (i.e. the bag energy or the false vacuum energy) $57 MeV/fermi^3 \times \frac{4}{3}\pi(1.0 fermi)^3$ (which is $238.8 MeV$). So, in the volume $1.0 cm^3$, we have at least $238.8 MeV \times 3.65 \times 10^{26}$ or $8.72 \times 10^{25} GeV$ in baryon bag energy. Or, in different units $8.72 \times 10^{25} / (0.5609 \times 10^{24}) gm/c^2$ or $155.5 gm/c^2$, compared to ρ_{vac} above ($\cong 10^{10} gm/cm^3$). Thus, only a tiny fraction of ρ_{vac} is to be hidden in baryons or other color-singlet objects after the QCD phase transition in the early Universe.

Where did the huge amount of the energy ρ_{vac} go? In the beginning of the end of the phase transition, the pasted domain walls with the huge kinetic energies seem to be the main story. A pasted domain wall is forming by colliding two domain walls while eliminating the false vacuum in between. The kinetic energies associated with the previously head-on collision become vibration, center-of-mass motion, etc. Of course, the pasted domain walls would evolve much further such as through the decaying interactions given earlier or forming the "permanent" structures. In any case, the total energy involved is known reasonably - a large fraction of ρ_{vac} , much larger than the radiation ρ_γ (with ρ_m negligible at this point).

Shown in Fig. 1 is the key result of the previous papers [5, 15] - we wish to use it to explain the important result. At $t \sim 3.30 \times 10^{-5} sec$, where did the latent energy $10^{14} gm/cm^3$ evolve into? We should know that the curve for ρ_γ , for massless relativistical particles, is the steepest in slope. The other curve for ρ_m is the other limit for matter (which $P \approx 0$). In this way, the latent energy is connected naturally with the curve for ρ_{DM} - in fact, there seems to be no other choice.

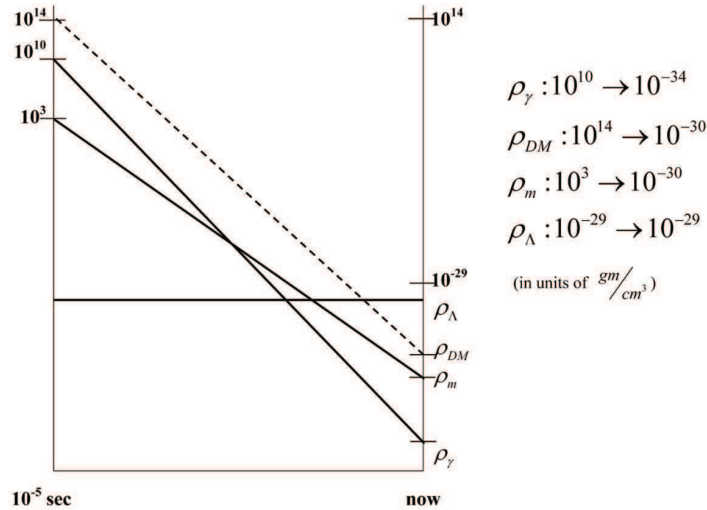


Figure 1: The various densities of our universe versus time.

Coming back to Eq. (2), we could assume for simplicity that when the cosmological QCD just took place the system follows with the relativistic pace (i.e. $P = \rho/3$) but when the system over-stretched enough and had evolved long enough it was diluted enough and

became non-relativistic (i.e. $P \approx 0$). It so happens that in both cases the density to the governing equation, Eq. (2), always looks like $\rho \propto t^{-2}$, although it is $R \propto t^{\frac{1}{2}}$ followed by $R \propto t^{\frac{2}{3}}$.

It is so accidental that what we call "the radiation-dominated universe" is in fact dominated by the latent energy from the cosmological QCD phase transition in the form of "pasted" or "patched" domain walls and the various evolved objects. In our case, the transition into the "matter-dominated universe", which happened at a time slightly different from $t \sim 10^9 \text{ sec}$, occurred when all the evolutions of the pasted domain walls ceased or stopped. In other words, it is NOT the transition into the "matter-dominated universe", as we used to think of.

In fact, the way of thinking of the "dark matter", or the majority of it, turns out to be very natural. Otherwise, where did the 25% content of our universe come from? Of course, one could argue about the large amount of the cosmological QCD phase transition, in terms of the largeness of the latent energy. The curves in Fig. 1 might make a lot of sense [5, 15].

The role of the latent heat(energy) in the cosmological phase transitions poses another fundamental question that deserves our further thoughts.

5 The Outlook

In this note, we argue that from the macroscopic scale (1 cm) to the microscopic scale (10^{-8} cm) the variables get reduced (elimination from momenta/coordinates to coordinates) and so further reduction comes from changing commutative coordinates to noncommutative coordinates when the quantum era (10^{-8} cm) has changed to the super-quantum era (10^{-20} cm or small). We realized that H. Snyder[10] as early as 1947 tried to extend the space-time to a discrete one while observing Lorentz symmetry - a nice formulation of our "super-quantum" idea. The practical calculations come from [14, 13]. The noncommutative idea comes from [10, 12]. Here we make links over the question of how smallest of the point-like particles really are.

We also have illustrated an approximate way to treat the cosmological QCD phase transition, a result that indicates the basic importance of the latent heat (energy) if the phase transition would be first-order in nature. In view of the complicate nature of the subject, we suspect that it will remain the unsettled case for a while (for many years or decades). In the case of EW phase transition, there is some progress but rather limited. In both cases, we still call them the "normal" phase transitions in terms of the matter densities involved (i.e. not yet the "super-quantum" regime).

We hope that our discussions would shed lights on the two fundamental questions.

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References

- [1] G. Smoot *et al.*, *Astrophys. J.* **396**, L1 (1992); C. Bennett *et al.*, *Astrophys. J.* **396**, L7 (1992); E. Wright *et al.*, *Astrophys. J.* **396**, L11 (1992).
- [2] C. L. Bennett, M. S. Turner, and M. White, *Physics Today*, November 1997, p. 32, for an early general review.
- [3] See, e.g., the news in *Physics Today*, April 2003, p. 21; and the references therein.
- [4] See, e.g., the news in *Physics Today*, May 2006, p.16; and the references therein.
- [5] W-Y. P. Hwang, *Internat'l J. Mod. Phys.* **A23**, 4757 (2008).
- [6] E.W. Kolb and M.S. Turner, *The Early universe*, *Addison-Wesley Publishing Co.* (1994).
- [7] S. Perlmutter *et al.* [Supernova Cosmology Project], *Astrophys. J.* **517**, 565 (1999); A. G. Riess *et al.* [Supernova Search Team], *Astron. J.* **116**, 1009 (1998).
- [8] R.R. Caldwell, R. Dave, and P.J. Steinhardt, *Phys. Rev. Lett.* **80**, 1582 (1998); Je-An Gu and W-Y. P. Hwang, *Phys. Lett.* **B517**, 1 (2001).
- [9] W-Y. P. Hwang, *International J. Mod. Phys.* **24**, 3366 (2009) [in *Proceedings of the Conference on Particle Physics, Astrophysics and Quantum Field Theory: 75 Years since Solvey*]; W-Y. Pauchy Hwang, in *CP1150, Frontiers in Physics - 3rd International Meeting*, eds. S.-P. Chia, M.R. Muhumud, and K. Ratnavelu @2009 American Institute of Physics 978-0-7354-0687-2/09, pp. 25-30, (2009).
- [10] H.S. Snyder, *Phys. Rev.* **71**, 38 (1947); **72**, 68 (1947).
- [11] Particle Data Group, *Review of Particle Physics*, *J. Phys. G: Nucl. Part. Phys.* **37**, 1 (2010).
- [12] A. Connes, *Non-Commutative Geometry* (Academic Press, New York, 1994); M. Li and T. Yoneya, *Phys. Rev. Lett.* **78**, 1219 (1997); C.-S. Chu and P.-M. Ho, *Nucl. Phys.* **B550**, 151 (1999); A. Connes, M.R. Douglas, A. Schwarz, *J. High Energy Phys.* **02**, 003 (1998); M.R. Douglas and C. Hull, *ibid.* **02**, 008 (1998); N. Seiberg and E. Witten, *ibid.* **09**, 032 (1999); M.M. Sheikh-Jabbari, *Phys. Lett.* **B455**, 129 (1999); J.L.F. Barbon and E. Rabinovici, *ibid.* **486**, 202 (2000); R. Gopakumar, J. Maldacena, S. Minwalla, and A. Strominger, *J. High Energy Phys.* **06**, 036 (2000); D. Bigatti and L. Susskind, *Phys. Rev.* **D62**, 066004 (2000); N. Seiberg, L. Susskind, and N. Toumbas, *J. High Energy Phys.* **06**, 021 (2000); D.J. Gross, A. Hashimoto, and N. Itzhaki, hep-th/0008075; T. Pengpan and X. Xiong, *Phys. Rev.* **D63**, 085012 (2001); F. J. Petriello, *Nucl. Phys.* **B601**, 169 (2001).
- [13] C.P. Martin and D. Sanchez-Ruiz, *Phys. Rev. Lett.* **83**, 476 (1999); M. Hayakawa, *Phys. Lett.* **B478**, 394 (2000); Ihab. F. Riad and M.M. Sheikh-Jabbari, *J. High Energy Phys.* **08**, 045 (2000); M. Chaichian, M.M. Sheikh-Jabbari, and A. Tureanu, *Phys. Rev. Lett.* **86**, 2716 (2001); H. Arfael and M.H. Yavartanoo, hep-th/0010244.

- [14] S. Baek, D.K. Ghosh, X.-G. He, and W-Y. P. Hwang, Phys. Rev. **D64**, 056001 (2001).
- [15] W-Y. P. Hwang, J. Korean Phys. Soc. **53**, 1115 (2008).
- [16] A. Chodos, R.L. Jaffe, K. Johnson, C.B. Thorn, and V.F. Weisskopf, Phys. Rev. **D9**, 3471 (1974); T.A. DeGrand, R.L. Jaffe, K. Johnson, and J. Kiskis, Phys. Rev. **D12**, 2060 (1975). For the bag model parameters, see, e.g., W-Y. P. Hwang, Phys. Rev. **D31**, 2826 (1985).
- [17] R. Friedberg and T.D. Lee, Phys. Rev. **D16**, 1096 (1977); **D18**, 2623 (1978). For generalizations, see, e.g., W-Y. P. Hwang, Phys. Lett. **116B**, 37 (1982) and Phys. Rev. **D29**, 1465 (1984).